

Correspondence

Parametric Amplification*

According to Manley-Rowe¹ power relations the difference of pump and signal frequencies yields the so-called negative resistance parametric amplifier. In this case, infinite gain is possible in contrast to the sum frequency amplifier or up-converter which has a finite gain determined by the pump and signal frequencies.

In order to realize parametric amplification, the circuit in question must contain a time varying or nonlinear reactive element which is usually accompanied by a nonlinear resistance such as the function of a semiconductor diode. Under this condition, the harmonic content of the resultant wave is very high, due to the existence of both amplitude and frequency modulation. Harmonic generators using this idea have been developed by Leenov and Uhrlir.² Another condition which results in combined amplitude and frequency modulation is the nondissipative *LC* circuit with a time varying capacitor. Such a circuit involves the solution of the Mathieu's³ equation given in (1)

$$q'' + \delta(1 + \cos W_s t)q = 0 \quad (1)$$

where

$$\delta = \frac{(\text{natural frequency})^2}{(\text{signal frequency})^2}.$$

The sideband expressions of an amplitude modulation in presence of frequency modulation are given by (2) and (3).⁴

$$I_{F-f} = 0.5K I_M (B_0 + B_2) \cos(\Omega - w)t - B_1 I_M \cos(\Omega - w)t \quad (2)$$

$$I_{F+f} = -0.5K I_M (B_0 + B_2) \cos(\Omega + w)t - B_1 I_M \cos(\Omega - w)t \quad (3)$$

where

Ω = carrier or pump frequency

w = signal frequency

$B_0 = J_0(\beta)$

$B_1 = J_1(\beta)$

$B_2 = J_2(\beta)$

Eqs. (2) and (3) show a very important fact. The amplitude of the lower sideband is the difference of two factors as given in (4).

$$0.5K[B_1 + B_2] - B_1 I_M = (c - d)I_M \quad (4)$$

whereas the upper sideband is the sum of two negative factors. Under this condition, the magnitude of the lower sideband can be a very small percentage of the upper sideband. Therefore, one can talk of a negative type of parametric amplification if the product of parametric amplification gain and the

amplitude of the lower sideband exceeds the amplitude of the upper sideband. Also, under very small lower sideband amplitudes, the signal-to-noise ratio may be a predominating factor in determining the quality of the amplifier. An important conclusion which can be drawn from the above explanation would be that up-converters may be superior to negative type of amplifiers in gain and noise considerations if the value of $(c - d)I_M$ goes to very low levels.

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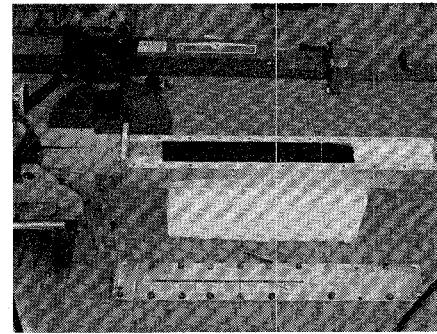


Fig. 1—Equipment for measurement of dielectric constant and loss tangent of soil.

A Microwave Technique for the Measurement of the Dielectric Properties of Soils*

During a recent investigation of antennas mounted flush with the earth, it was necessary to measure the dielectric properties of soil. Many different techniques for the measurement of dielectric constants and loss tangent have been developed.¹⁻² These techniques generally fall into two categories: those which utilize transmission through a sample and those which use the reflection from the sample. The particular technique employed is modified to suit the range of parameters being measured and the physical characteristics of the sample. In general, a measurement that provides an accurate determination of dielectric constant is relatively insensitive to variations in loss tangent and vice versa. The technique described here utilizes both types of measurements to find in a practical manner the dielectric constant and loss tangent of relatively moist loamy soil at microwave frequencies.

The attenuation of a wave propagating in the sample is measured by probing a soil-filled section of rectangular waveguide through a series of holes drilled along the centerline of the top face. A typical experimental setup is shown in Fig. 1 where the top face of the waveguide has been removed to show the sample of soil. The effect of the reflected wave in the soil may be neglected by making the sample sufficiently long. The magnitude of the field is then given by

$$E = E_0 e^{-(\alpha/k_0)x} \quad (1)$$

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¹ J. M. Manley and H. E. Rowe, "Some general properties of nonlinear elements, Part 1: General energy relations," *PROC. IRE*, vol. 44, pp. 904-913; July, 1956.

² D. Leenov and A. Uhrlir, "Generation of harmonics and subharmonics at microwave frequencies with *P-N* junction diodes," *PROC. IRE*, vol. 47, pp. 1724-1729; October, 1959.

³ J. R. Carson, "Notes on the theory of modulation," *PROC. IRE*, vol. 10, pp. 57-64; February, 1922.

⁴ A. Hund, "Frequency Modulation," McGraw-Hill Book Co., Inc., New York, N. Y., p. 63, 1942.

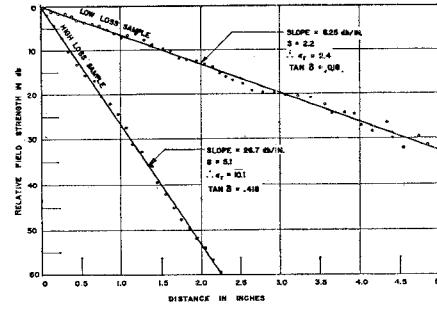


Fig. 2—Attenuation in waveguide filled with two different samples of soil.

where α is attenuation constant, k_0 is the free space propagation constant (*i.e.*, $k_0 = 2\pi/\lambda_0$ where λ_0 is free space wavelength), x is the distance measured along the waveguide, and E_0 is the magnitude of the field at the front edge of the sample. Thus, if the magnitude of the field in db is plotted against distance along the guide, the result is a straight line whose slope is proportional to α/k_0 . Fig. 2 shows examples of probe data taken with two different samples of soil. It will be observed that there are slight variations from the straight line curve which are due largely to nonuniform density of the soil sample; however, with a sufficient number of points, the slope of the line is quite accurately determined. No essential variation from the straight line was observed with a homogeneous material such as soap.

The propagation constant (γ) for the dominant mode in the soil filled portion of the rectangular waveguide may be written (assuming $e^{j\omega t}$ time dependence) as

$$\gamma = \alpha + j\beta = k_0 \left[\left(\frac{\lambda_0}{\lambda_e} \right)^2 - \epsilon_r + j\epsilon_r \tan \delta \right]^{1/2} \quad (2)$$

where ϵ_r is the relative dielectric constant of the soil, $\tan \delta$ is the loss tangent of the soil, and λ_e is equal to twice the waveguide width. Solving for the loss tangent gives

$$\tan \delta = \frac{2}{\epsilon_r} \left(\frac{\alpha}{k_0} \right) \left[\left(\frac{\alpha}{k_0} \right)^2 + \epsilon_r - \left(\frac{\lambda_0}{\lambda_e} \right)^2 \right]^{1/2}. \quad (3)$$

This shows that the attenuation constant is sensitive to changes in loss tangent.

The normalized impedance z in the air-filled portion of waveguide at the interface of the soil sample is equal to the ratio of the propagation constant in the air-filled guide to that in the soil-filled guide and is thus written as

$$z = \frac{\gamma_0}{\gamma} = \left[\frac{1 - \left(\frac{\lambda_0}{\lambda_c} \right)^2}{\epsilon_r(1 - j \tan \delta) - \left(\frac{\lambda_0}{\lambda_c} \right)^2} \right]^{1/2}. \quad (4)$$

In turn, the VSWR S in the air-filled guide is related to z by

$$S = \frac{|z+1| + |z-1|}{|z+1| - |z-1|}. \quad (5)$$

Substitution of (3) and (4) in (5) yields an expression for S containing ϵ_r , the measured quantity α/k_0 and waveguide parameters. For the special case when $\epsilon_r \geq 2$ and $\tan \delta \leq 0.1$, (5) simplifies to

$$S \approx \frac{1}{z} \approx \left[\frac{\epsilon_r - \left(\frac{\lambda_0}{\lambda_c} \right)^2}{1 - \left(\frac{\lambda_0}{\lambda_c} \right)^2} \right]^{1/2}. \quad (6)$$

This shows that measurement of VSWR is sensitive to changes in dielectric constant.

Eqs. (3), (4) and (5) are plotted in Figs. 3 and 4 as a function of α/k_0 for an operating frequency of 8.6 kMc. Thus, for a given measured α/k_0 and S , the corresponding dielectric constant may be read from Fig. 3 and then the value of $\tan \delta$ may be

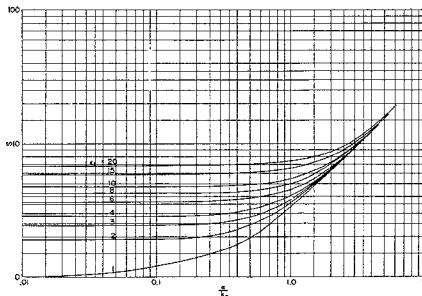


Fig. 3—VSWR as a function of α/k_0 for $f=8.6$ kMc.

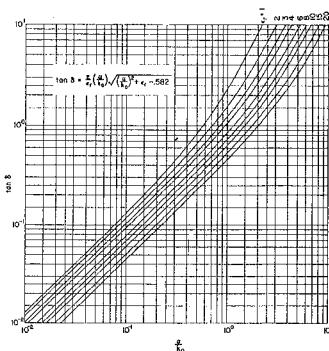


Fig. 4—Loss tangent as a function of α/k_0 for $f=8.6$ kMc.

taken from Fig. 4. Not only has this technique proved useful for measuring the properties of soil, but it has also been used with great success in a bio-medical research program for measuring the properties of fatty tissue at L - and S -band frequencies.

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and the input signal frequency is in frequency range of oscillation of the amplifier, it is difficult to control the oscillation to obtain stable amplification. On the other hand if the signal frequency is just outside the oscillation region and the impedances and voltages are such that oscillation does not exist, stable amplification is easily obtained.

Thus it can be seen that the output impedance of the amplifier had a great effect on its performance. As an example, with a -70 dbm 70.35 kMc input signal to the VA-99, strong oscillation appeared approximately 30 Mc off the signal. When one of the EH tuners adjacent to the VA-99 was adjusted, the oscillation weakened as it moved to within 18 Mc of the signal which was now being amplified. By further adjustment of the EH tuner, oscillation disappeared near the signal frequency and the signal alone remained, amplified 27 db.

When the VA-99 was oscillating strongly

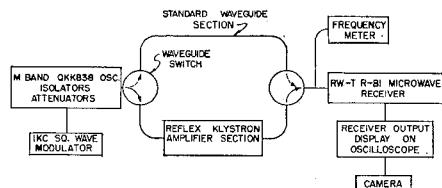


Fig. 1—Test circuit configuration.

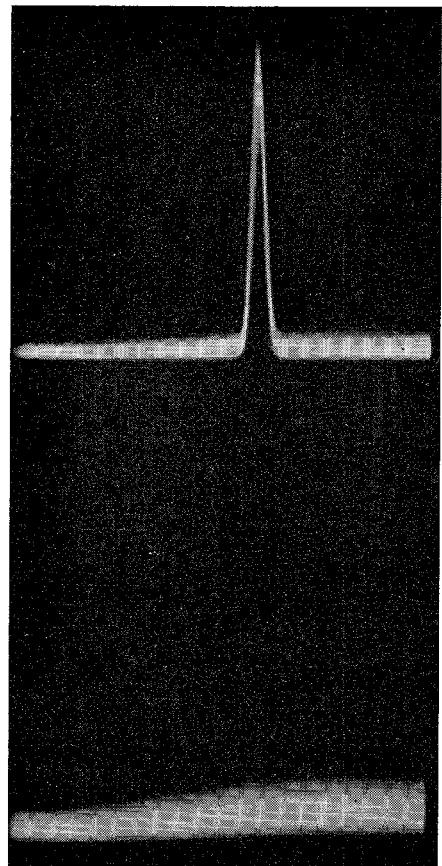


Fig. 2—Pulse amplification (frequency sweep display). Top: Output of VA-99 Amplifier, 30 db gain 70.35 kMc carrier. Bottom: 1-μsec pulse input to VA-99 amplifier, pulse repetition rate, 1000 cps.

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³ K. Ishii, "Using reflex klystron as millimeter wave amplifiers," *Electronics*, vol. 33, pp. 71-73; March, 1960.

⁴ D. M. Makurat, R. C. Hertel and K. Ishii, "The reflex klystron as an amplifier at 73 kMc," *PROC. IRE (Correspondence)* vol. 50, pp. 210-211; February, 1962.

⁵ K. Ishii, D. E. Schumacher and K. R. Kelly, "Interesting behavior of VA-99 as a millimeter wave amplifier," *PROC. IRE (Correspondence)* vol. 50, p. 2510; December, 1962.

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